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Digital Adaptive Control

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1988 REPORT ON PROJECT

DIGITAL ADAPTIVE CONTROL

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Bilateral Cooperation between

TECHNISCHE HOCHSCHULE DARMSTADT
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and

UNIVERZA "EDVARDA KARDELJA" LJUBLJANA
FAKULTETA ZA ELEKTROTEHNIKO IN RAČUNALNIŠTVO
LABORATORIJ ZA ANALOGNO–HIBRIDNO RAČUNANJE IN
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CONTENT

FOREWORD

In the frame of bilateral cooperation between Federal republic Germany and Yugoslavia, which is being supported by the "International Bureau of KFA Jülich", a common project entitled "Digital adaptive control" was started in 1987.

The working partners in this cooperation are "Institut für Regelungstechnik, Fachgebiet Regelsystemtechnik" at the "Technische Hochschule Darmstadt" and "Fakulteta za elektrotehniko in računalništvo, Laboratorij za analogno-hibridno računanje in avtomatsko regulacijo at the "Univerza Edvarda Kardelja v Ljubljani".

In the 1987 Report on the project the recursive parameter estimation methods and the model reference adaptive control systems were presented.

According to the working program in the second year of the research different control algorithms for parameter-adaptive control systems were further developed and are presented in Chapter 1 of this Report. Chapter 2. emphasises the monitoring and coordination level of parameter - adaptive control systems and in Chapter 3 comparisons of different parameter - adaptive controllers were performed by means of simulation. Parameter - adaptive control systems were implemented on a microcomputer and a hydraulic pilot plant was controlled. First results in this adaptive control scheme are presented in Chapter 4.

We would like to express our great gratitude to "International Bureau of KFA Jülich" for making this research possible.

Authors

Darmstadt, march 22 1989

1. CONTROL ALGORITHMS FOR PARAMETER-ADAPTIVE CONTROL SYSTEMS

1.1 Minimum variance controller

The goal of the minimum variance controller design is to minimize the variance of the controlled variable

$$I_1 = \text{var}\,[y(k)] = E\left\{y^2(k)\right\} \tag{1.1-1}$$

It is assumed that the process is disturbed by the noise n(k) as shown in the Fig.1.1.

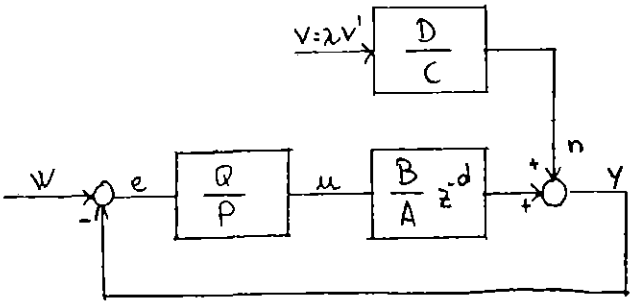

Fig. 1.1: Control of the process disturbed by stochastic disturbances

The noise n(z) is supposed to be a stationary random process and as shown in Åström, Wittenmark (1984) all stationary processes can be thought of as being generated by stable nonminimal phase linear systems driven by white noise, so n(k) is supposed to be a filtered gaussian white noise

$$n(z) = \frac{D(z^{-1})}{C(z^{-1})}\lambda v'(z) = \frac{D(z^{-1})}{C(z^{-1})} v(z) = \frac{1 + d_1 z^{-1} + \ldots + d_m z^{-m}}{1 + c_1 z^{-1} + \ldots + c_m z^{-m}} v(z) \tag{1.1-2}$$

λ in Eq. (1.1-2) is used to normalize the equation so that the polynomials $D(z^{-1})$ and $C(z^{-1})$ are monic. $v'(z)$ is a gaussian white noise with expectation $\bar{v}' = 0$ and variance $\sigma_{v'}^2 = 1$ and $v(z)$ is a gaussian white noise with expectation $\bar{v} = 0$ and variance $\sigma_v^2 = \lambda^2$. Due to the assumption of stationary random processes the zeros of $C(z^{-1})$ are assumed to be inside the unit circle. If drifting disturbances are considered, the zeros of $C(z^{-1})$ may be on the unit circle. Although the zeros of $D(z^{-1})$ are assumed to be inside or on the unit circle. If this is not the case, the polynomial $D(z^{-1})$ may be changed by spectral factorisation so that its zeros are inside or on the unit circle (Åström, Wittenmark, 1984).

The minimum variance control law using the criterion (1.1-1) and assuming the denominator of the noise filter to be equal to the denominator of the process transfer function $C(z^{-1}) = A(z^{-1})$ was developed by Åström (1970). The criterion (1.1-2) does not consider the manipulated variable $u(k)$ so in many cases excessive input changes are produced . As shown later the corresponding minimum variance controller is in its originally form applicable only to the processes with nonminimum phase behaviour. Clarke and Hasting - James (1971) proposed a weighting r of the manipulated variable. In this case the criterion

$$I_2 = E\left\{y^2(k+d+1) + r\, u^2(k)\right\} \qquad (1.1\text{-}3)$$

is minimized. The variance of the controlled variable is in this case not longer minimal; instead the variance of a combination of the controlled variable and the manipulated variable is minimal. The resulting generalized minimum variance control law is called also linear quadratic gaussian (LQG) control.

In this section the generalized minimum variance control laws for processes without the time delay and with it are given first and then its properties, applicability, and relations to other controllers are given.

Four types of the controller will be reviewed for chosen criterions (1.1-1) and (1.1-3) and for assumption $C(z^{-1}) = A(z^{-1})$ to be valid or not.

The generalized minimum variance controller for the criterion (1.1-3) - $r \neq 0$ and using noise model (1.1-2) for processes without time delay is given by (Isermann, 1981, 1987)

$$G_{RMV1}(z) = \frac{u(z)}{e(z)} = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{A(z^{-1})\,[\,D(z^{-1}) - C(z^{-1})\,]\,z}{zB(z^{-1})\,C(z^{-1}) + \frac{r}{b_1}\,A(z^{-1})\,D(z^{-1})} \tag{1.1-4}$$

(Abbreviation: MV1). This controller contains the process model with polynomials $A(z^{-1})$ and $B(z^{-1})$ and the noise model with polynomials $C(z^{-1})$ and $D(z^{-1})$. With $r = 0$, the minimum variance controller for the noise model (1.1-2) is produced

$$G_{RMV2} = \frac{A(z^{-1})\,[\,D(z^{-1}) - C(z^{-1})\,]\,z}{zB(z^{-1})C(z^{-1})} = \frac{zA(z^{-1})}{zB(z^{-1})}\left[\frac{D(z^{-1})}{C(z^{-1})} - 1\right] \tag{1.1-5}$$

(Abbreviation: MV2).

Under the assumption of the noise filter and process transfer function denominators equality $C(z^{-1}) = A(z^{-1})$ the controller

$$G_{RMV3} = \frac{[\,D(z^{-1}) - A(z^{-1})\,]\,z}{z\,B(z^{-1}) + \frac{r}{b_1}\,D(z^{-1})} \tag{1.1-6}$$

(Abbreviation: MV3) and for $r = 0$ the original minimum variance controller

$$G_{RMV4} = \frac{[\,D(z^{-1}) - A(z^{-1})\,]\,z}{z\,B(z^{-1})} \tag{1.1-7}$$

(Abbreviation: MV4) are obtained.

In the closed loop the disturbance influences the controlled variable through the dynamic control factor which for the MV1 is given by

$$R(z) = \frac{y(z)}{n(z)} = \frac{1}{1+G_R(z^{-1})G_p(z^{-1})} = \frac{zB(z^{-1})C(z^{-1}) + \frac{r}{b_1} A(z^{-1})D(z^{-1}}{[\frac{r}{b_1} A(z^{-1}) + z\ B(z^{-1}]\ D(z^{-1})} \tag{1.1-8}$$

For r = 0, i. e. for the controller MV2 the dynamic control factor

$$R(z) = \frac{C(z^{-1})}{D(z^{-1})} \tag{1.1-9}$$

becomes the inverse of the noise filter. The minimum variance control laws MV2 and MV4 force the close loop to behave as the reciprocal of the noise filter and so force the controlled variable to become white noise. This can be explained in a simple way as follows: Supposing w=0 the controlled variable is produced from the white noise v(k) according to the relation

$$\frac{y(z)}{v(z)} = \frac{D(z^{-1})}{C(z^{-1})} R(z) = G_v(z) \ . \tag{1.1-10}$$

The variance of the controlled variable y is now given by

$$\text{var}\ [y(k)] = \lambda^2\ [\ 1 + g_v^2(1) + g_v^2(2) + \dots\ ] \tag{1.1-11}$$

where $g_v(i)$ is the impulse response of the transfer function $G_v(z)$, see Åström (1970). It is obvious, that the variance is minimal if $g_v(1) = g_v(2) = \dots = 0$, i. e. if $H_v(z) = 1$. Thus the dynamic control factor R(z) must cancel the noise transfer function $\frac{D(z^{-1})}{C(z^{-1})}$.

Before the generalized minimum variance controller for processes with time delay will be reviewed another interpretation of the MV2 controller will be given. Due to this interpretation MV2 controller makes an one step prediction of the noise signal n(k):

$$n(k+1) = \frac{D(q^{-1})}{C(q^{-1})}\, v(k+1) = \frac{[D(q^{-1}) - C(q^{-1})]\, q}{C(q^{-1})}\, v(k) + v(k+1) =$$

$$= \frac{[\, D(q^{-1}) - C(q^{-1})\,]\, q}{C(q^{-1})}\, y(k) + v(k+1) \qquad (1.1\text{-}12)$$

where q^{-1} is the backward shift operator. The first term in Eq. (1.1-12) is completely known at the moment k, the second term in this equation is totally random and the best prediction of the noise signal n(k+1) is obtained if v(k+1) is replaced by its mean value, i. e. by 0. Now the manipulated variable u(k) is determined in such a way that the undisturbed process output compensates the disturbance n(k+1)

$$\frac{B(q^{-1})\, q}{A(q^{-1})}\, u(k) + \frac{[D(q^{-1}) - C(q^{-1})]q}{C(q^{-1})}\, y(k) = 0 \qquad (1.1\text{-}13)$$

yielding the control law

$$\frac{u(k)}{e(k)} = - \frac{u(k)}{y(k)} = \frac{A(q^{-1})\, [D(q^{-1}) - C(q^{-1})]\, q}{q\, B(q^{-1})\, C(q^{-1})} \qquad (1.1\text{-}14)$$

i.e the controller MV2.

Using this interpretation the minimum variance controller for processes with time delay will now be developed in a rather heuristic way. Due to the time delay of the process the manipulated variable in the moment k influences the controlled variable at the moment k+d+1, so for the compensation of the disturbance signal its (d+1) step prediction of the disturbance signal n is required. In order to obtain a causal predictor, the disturbance filter must be separated in two parts (Åström, Wittenmark, 1984)

$$\frac{D(z^{-1})}{C(z^{-1})} = F(z^{-1}) + \frac{L(z^{-1})}{C(z^{-1})} z^{-(d+1)} \tag{1.1-15}$$

where

$$F(z^{-1}) = 1 + f_1 z^{-1} + \ldots + f_d z^{-d} \tag{1.1-16}$$

and

$$L(z^{-1}) = l_o + l_1 z^{-1} + \ldots l_{m-1} z^{-(m-1)} \tag{1.1-17}$$

are two polynomials, to be determined from the identity

$$D(z^{-1}) = C(z^{-1}) F(z^{-1}) + L(z^{-1}) z^{-(d+1)} \tag{1.1-18}$$

which follows immediately from Eq. (1.1-15). The (d+1) step prediction of the disturbance can now be written in the following form

$$n(k+d+1) = \frac{D(q^{-1})}{C(q^{-1})} v(k+d+1) = \frac{L(q^{-1})}{C(q^{-1})} v(k) + F(q^{-1}) v(k+d+1) \ . \tag{1.1-19}$$

Again the first term in Eq. (1.1-19) is completely known at the moment k, since v(k) can be calculated from past values of the controlled and manipulated variables

$$v(k) = \frac{C(q^{-1})}{D(q^{-1})} n(k) = \frac{C(q^{-1})}{D(q^{-1})} \left[y(k) - \frac{B(q^{-1}) q^{-d}}{A(q^{-1})} u(k) \right] \ . \tag{1.1-20}$$

Introducing Eq (1.1-20) into Eq. (1.1-19) yields

$$n(k+d+1) = \frac{L(q^{-1})}{D(q^{-1})} \left[y(k) - \frac{B(q^{-1})}{A(q^{-1})} q^{-d} u(k) \right] + F(q^{-1}) v(k+d+1) \ . \tag{1.1-21}$$

The term $F(q^{-1})\ v(k+d+1)$ is completely random at the moment k and the best prediction of $n(k+d+1)$ i.e. $n(k+d+1/k)$ is obtained if it is replaced by its mean value, i.e. by 0. The manipulated variable at the moment k i. e. $u(k)$ is determined again in such a way that the undisturbed process output $y_u(k+d+1)$ compensates the predicted disturbance at the moment k+d+1 and sets the predicted process output $y(k+d+1/k)$ to zero

$$y(k+d+1/k) = y_u(k+d+1) + n(k+d+1/k) = 0 =$$

$$= \frac{B(q^{-1})q^{-d}}{A(q^{-1})} q^{(d+1)} u(k) + \frac{L(q^{-1})}{D(q^{-1})} \left[y(k) - \frac{B(q^{-1})}{A(q^{-1})} q^{-d}\ u(k) \right] . \tag{1.1-22}$$

Using this equation and identity (1.1-18) the control law

$$\frac{u(k)}{e(k)} = - \frac{u(k)}{y(k)} = \frac{A(q^{-1})\ L(q^{-1})}{q\ B(q^{-1})\ C(q^{-1})\ F(q^{-1})} \tag{1.1-23}$$

is obtained. So the transfer function of the minimum variance controller for the processes with time delay and using the criterion (1.1-1) is the following

$$G_{RMV2d} = \frac{A(z^{-1})\ L(z^{-1})}{z\ B(z^{-1})\ C(z^{-1})\ F(z^{-1})} \tag{1.1-24}$$

(Abbreviation: MV2-d)

The generalized minimum variance controller for processes with time delay is given by

$$G_{RMV1d}(z) = \frac{A(z^{-1})\ L(z^{-1})}{z\ B(z^{-1})\ C(z^{-1})\ F(z^{-1}) + \frac{r}{b_1} A(z^{-1})\ D(z^{-1})} \tag{1.1-25}$$

(Abbreviation: MV1-d),

see Isermann (1984, 1987)

The minimum variance controller for r=0 is given by Eq. (1.1-24). With $C(z^{-1}) = A(z^{-1})$ follow the controllers

$$G_{RMV3d}(z) = \frac{L(z^{-1})}{z\ B(z^{-1})\ F(z^{-1}) + \frac{r}{b_1}\ D(z^{-1})} \tag{1.1-26}$$

(Abbreviation: MV3-d) for $r \neq 0$ and

$$G_{RMV4d}(z) = \frac{L(z^{-1})}{z\ B(z^{-1})\ F(z^{-1})} \tag{1.1-27}$$

(Abbreviation MV4-d) for $r = 0$.

Now the properties of all eight controllers will be discussed. The most general controller is MV1-d and all controllers represent a subset of it. The controllers for the processes without the time delay are obtained by introducing $d = 0$ into the controller equation. The identity (1.1-18) yields for this case

$$\begin{aligned} F(z^{-1}) &= 1 \\ L(z^{-1}) &= z\ [\ D(z^{-1}) - C(z^{-1})\ ]\ . \end{aligned} \tag{1.1-28}$$

The controller MV2-d is obtained from the controller MV1-d by using $r = 0$, the controller MV3 by supposing $C(z^{-1}) = A(z^{-1})$ and the controller MV4-d by introducing $r = 0$ and $C(z^{-1}) = A(z^{-1})$. The properties of all eight controllers may be summarized as follows

a) Controller order

	Numerator	Denominator
MV1:	2m-1	2m
MV1-d:	2m-1	2m+d-1
MV2:	2m-1	2m-1
MV2-d:	2m-1	2m+d-1
MV3:	m-1	m
MV3-d:	m-1	m-1
MV4:	m-1	m-1
MV4-d:	m-1	m+d-1

b) Cancellation of poles and zeros

MV1, MV1-d: The controller cancels the poles ($A(z^{-1}) = 0$) and though can not be applied to unstable or poor stable processes.

MV2, MV2-d: Both, the process poles and zeros are canceled and therefore the controller should not be applied neither to unstable (and poor stable) processes nor to the processes with nonminimum phase behaviour.

MV3, MV3-d: No cancellations and though in general no restrictions

MV4, MV4-d: The process zeros ($B(z^{-1}) = 0$) are canceled and thus the controller is not applicable to processes with nonminimum phase behaviour.

c) Stability

The characteristic equation of the closed loop

$$1 + G_P(z)\, G_{RMV1d}(z) = 0 \qquad (1.1\text{-}29)$$

becomes for the MV1-d controller with the use of Diophantine eqation and the identity (1.1-18) as follows

$$A(z^{-1})\ D(z^{-1}) \left[z\ B(z^{-1}) + \frac{r}{b_1} A(z^{-1}) \right] = 0 \qquad (1.1\text{-}30)$$

The same equation is obtained for the MV1 controller. As the process transfer function denominator $A(z^{-1})$ appears in the characteristic equation of the closed loop, it is obvious that the MV1 and MV1-d controllers are applicable only to asymptotically stable processes; the same restriction as that one following from b).

If stationary disturbances are assumed, the zeros of the polynomial $D(z^{-1})$ can not lie outside the unit circle, but in order to obtain globally stable closed loop behaviour they must not lie on the unit circle either.

For a stable closed loop behaviour also the zeros of

$$\left[z\ B(z^{-1}) + \frac{r}{b_1} A(z^{-1}) \right] = 0 \qquad (1.1\text{-}31)$$

must lie within the unit circle. For the MV2 and MV2-d controllers (r=0) the zeros of Eq. (1.1-31) become the zeros of the process transfer function numerator $B(z^{-1})$. Though the MV2 and MV2-d controllers can not be applied to the processes with nonminimum phase behaviour. The larger the weight r on the manipulated variable, the nearer are zeros of the term (1.1-31) to the zeros of $A(z^{-1}) = 0$, i.e. to the process transfer function denominator poles.

The characteristic equation of the closed loop using MV3 and MV3-d controllers is

$$D(z^{-1}) \left[z\ B(z^{-1}) + \frac{r}{b_1} A(z^{-1}) \right] = 0 \qquad (1.1\text{-}32)$$

In this equation the process transfer function denominator $A(z^{-1})$ does not appear and though the assumption of stable processes is not needed any more. Other restrictions, especially those concerning the zeros of the polynomial $B(z^{-1})$ must be considered.

It should be pointed out that minimum variance controllers without weighting of the manipulated variable ($r = 0$) can be designed also for processes with nonminimum phase. As shown in Åström, Wittenmark, (1984) the variance of the controlled variable may have several local minima if the polynomial $B(z^{-1})$ has zeros outside the unit circle. The absolute minimum (corresponding to the controllers MV2, MV2-d, MV4 or MV4-d respectively) serves an unstable control due to the unbounded control variable, but there exists a local minima which serves a stable close loop behaviour.

The MV1, MV1-d, MV2 and MV2-d controllers can be designed for unstable processes too, Åström, Wittenmark, (1984). In this case the equation of the disturbed process

$$y(z) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} u(z) + \frac{D(z^{-1})}{C(z^{-1})} v(z) \tag{1.1-33}$$

is written in the form

$$A(z^{-1})\ C(z^{-1})\ y(z) = B(z^{-1})\ C(z^{-1})\ u(z) + A(z^{-1})\ D(z^{-1})\ v(z)\ . \tag{1.1-34}$$

Then the term $A(z^{-1})\ D(z^{-1})$ is changed by spectral factorisation so that all his zeros are inside or on the unit circle. The resulting controller cancels only stable zeros of the polynomial $A(z^{-1})$.

d) Dynamic control factor and controlled variable

The dynamic control factor for the MV1-d controller is given by the following equation

$$R(z) = \frac{y(z)}{n(z)} = \frac{z\,B(z^{-1})\;C(z^{-1})\;F(z^{-1}) + \frac{r}{b_1}\,A(z^{-1})\;D(z^{-1})}{[\,\frac{r}{b_1}\,A(z^{-1}) + z\,B(z^{-1})\,]\;D(z^{-1})} \tag{1.1-35}$$

For MV2 follows

$$R(z) = \frac{C(z^{-1})}{D(z^{-1})}\,F(z^{-1}) = 1 - \frac{L(z^{-1})}{D(z^{-1})}\,z^{-(d+1)} \tag{1.1-36}$$

The reciprocal of the disturbance filter arises again in the dynamic control factor but it is in the case $d > 0$ multiplied by the polynomial $F(z^{-1})$. The controlled variable becomes in this case

$$y(z) = R(z)\,\frac{D(z^{-1})}{C(z^{-1})}\,v(z) = F(z^{-1})\;v(z) \tag{1.1-37}$$

i. e. a moving average process

$$y(k) = v(k) + f_1 v(k-1) + \ldots + f_d v(k-d) \tag{1.1-38}$$

which corresponds to the unpredictable part of the disturbance prediction $n(k+d+1)$ in Eq.(1.1-19). The variance of the controlled variable is given by

$$\mathrm{var}[y(k)] = E(y^2(k)) = 1 + f_1^2 + \ldots + f_d^2 \tag{1.1-39}$$

and the larger the dead time the larger is the variance of the controlled variable.

e) Relations with other controllers

Taking into account the characteristic equation of the closed loop (Eq.(1.1-30)) the generalized minimum variance controller MV1-d can be interpreted in terms of the pole placement design as follows: The closed loop characteristic equation

$$\mathcal{A}(z^{-1}) = A(z^{-1})\ D(z^{-1})\ [\ z\ B(z^{-1}) + \frac{r}{b_1}\ A(Z^{-1})\ ] \qquad (1.1\text{-}40)$$

has its poles at

- the poles of the process transfer function denominator $A(z^{-1})$
- the poles of the noise filter transfer function numerator $D(z^{-1})$
- the zeros of the expression

$$[\ z\ B(z^{-1}) + \frac{r}{b_1}\ A(Z^{-1})\ ] = 0\ . \qquad (1.1\text{-}31)$$

With $r = 0$ the zeros of Eq.(1.1-38) becomes the zeros of the process transfer function numerator and with increasing r they tend to the process poles. So the minimum variance controller is a linear controller with the prescribed poles given by Eq.(1.1-40).

For the MV3-d controller the closed loop characteristic polynomial becomes

$$\mathcal{A}(z^{-1}) = D(z^{-1})\ [\ z\ B(z^{-1}) + \frac{r}{b_1}\ A(Z^{-1})\ ] \qquad (1.1\text{-}41)$$

and for the MV4-d

$$\mathcal{A}(z^{-1}) = D(z^{-1})\ z\ B(z^{-1}) \qquad (1.1\text{-}42)$$

The relations between MV4-d controller and the pole placement design become even more clear if the basic equation of the pole placement design

$$\mathcal{A}(z^{-1}) = P(z^{-1})\ A(z^{-1}) + Q(Z^{-1})\ B(z^{-1})\ z^{-d} \qquad (1.1\text{-}43)$$

is rewritten using Eq.(1.1-42) into the following form

$$D(z^{-1})\ z\ B(z^{-1}) = P(z^{-1})\ A(z^{-1}) + Q(z^{-1})\ z\ B(z^{-1})\ z^{-(d+1)} \qquad (1.1\text{-}44)$$

Obviously the term $z\ B(z^{-1})$ must be a part of the polynomial $P(z^{-1})$

$$P(z^{-1}) = P'(z^{-1})\ z\ B(z^{-1}) \tag{1.1-45}$$

yielding

$$D(z^{-1}) = P'(z^{-1})\ A(z^{-1}) + Q(z^{-1})\ z^{-(d+1)} \tag{1.1-46}$$

This equation corresponds to the identity (1.1-18) for the MV4-d controller ($C(z^{-1}) = A(z^{-1})$) with $P'(z^{-1}) = F(z^{-1})$ and $Q(z^{-1}) = L(z^{-1})$. With these identities the general linear control law becomes

$$G_R(z) = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{L(z^{-1})}{F(z^{-1})\ z\ B(z^{-1})} \tag{1.1-47}$$

i.e. the control law MV4-d.

Now the relations between the minimum variance controllers and the cancellation controllers will be discussed. The closed loop transfer function $G_w(z)$ using the MV1-d controller is given by

$$G_w(z) = \frac{G_R(z)\ G_p(z)}{1 + G_R(z)\ G_p(z)} = \frac{L(z^{-1})\ \ B(z^{-1})\ z^{-d}}{z\ B(z^{-1})\ D(z^{-1}) + \frac{r}{b_1}\ A(z^{-1})\ D(z^{-1})} \tag{1.1-48}$$

for the MV4-d controller this equation becomes

$$G_w(z) = \frac{L(z^{-1})}{D(z^{-1})}\ z^{-(d+1)} = \frac{D(z^{-1}) - A(z^{-1})\ F(z^{-1})}{D(z^{-1})} = \frac{\mathcal{B}_m(z^{-1})}{\mathcal{A}_m(z^{-1})} \tag{1.1-49}$$

From this equation it is obvious that

$$\mathcal{A}_m(z^{-1}) = D(z^{-1}) \tag{1.1-50}$$

$$\mathcal{B}_m(z^{-1}) = D(z^{-1}) - A(z^{-1})\ F(z^{-1}) \qquad (1.1\text{-}51)$$

and

$$\mathcal{A}_m(z^{-1}) - \mathcal{B}_m(z^{-1}) = A(z^{-1})\ F(z^{-1}) \ . \qquad (1.1\text{-}52)$$

The MV4-d controller can though be interpreted also as an cancellation controller with prescribed model transfer function (1.1-49). It cancels all the zeros of the process transfer function, but it cancels no poles because the term $[\mathcal{A}_m(z^{-1}) - \mathcal{B}_m(z^{-1})]$ includes all process poles, comp. Eq. (1.1-52) and the discussion in sect. 1.2 in Matko, Schwamberger 1987.

The MV4-d controller, designed originally for stochastic disturbance elimination, represents also an alternative to the deadbeat controller, designed for deterministic disturbance elimination. The deadbeat controller cancels all poles of the process transfer function, the MV4-d controller all the zeros. So the deadbeat controller may not be applied to unstable processes, the MV4-d controller not to the processes with nonminimum phase behaviour respectively. The desired input - output transfer function numerator $\mathcal{B}_m(z^{-1})$ of the deadbeat controller includes all process zeros, the difference polynomial $\mathcal{A}_m(z^{-1}) - \mathcal{B}_m(z^{-1})$ of the desired input - output transfer function for the MV4-d controller all process poles. So the deadbeat controllers may be applied to the processes with nonminimum phase behaviour and the MV4-d controller to unstable processes respectively, comp. sect 1.2 in Matko, Schwamberger 1987

f) Extensions

Clarke and Gawthrop (1975,1979) developed the generalized minimum variance controller which minimizes the cost function

$$I_3 = E\left\{[y_a(k+d+1) - w_a(k)]^2 + u_a^2(k)\right\} \qquad (1.1\text{-}53)$$

where y_a, u_a and w_a are auxiliary process output

$$y_a(z) = \frac{R_n(z^{-1})}{R_d(z^{-1})}\, y(z) \ , \tag{1.1-54}$$

auxiliary process input

$$u_a(z) = R_u(z^{-1})\; u(z) = (r_o + r_1 z^{-1} + \ldots + r_r z^{-r})\; u(z) \tag{1.1-55}$$

and auxiliary reference signal

$$w_a = R_w(z^{-1})\; w \tag{1.1-56}$$

respectively. $R_n(z^{-1})$, $R_d(z^{-1})$, $R_u(z^{-1})$ and $R_w(z^{-1})$ are polynomials to be chosen by the designer. In this case the prediction of the noise component in the auxiliary process output

$$n_a(k+d+1) = \frac{R_n(q^{-1})\; D(q^{-1})}{R_d(q^{-1})\; C(q^{-1})}\, v(k+d+1) \tag{1.1-57}$$

is obtained using the modified identity

$$\frac{R_n(z^{-1})\; D(z^{-1})}{R_d(z^{-1})\; C(z^{-1})} = F(z^{-1}) + \frac{L(z^{-1})}{R_d(z^{-1})\; C(z^{-1})}\, z^{-(d+1)} \tag{1.1-58}$$

in the following form

$$n_a(k+d+1) = \frac{L(q^{-1})}{R_d(q^{-1})\; C(q^{-1})}\, v(k) + F(q^{-1})\; v(k+d+1) \tag{1.1-59}$$

The last term in this equation is completely random again; the first term can be determined from the past values of the controlled and manipulated variables according to Eq.(1.1-20). Optimal prediction of the auxiliary output is then determined by the following expression

$$y_a(k+d+1/k) = \frac{R_n(q^{-1})}{R_d(q^{-1})} \frac{B(q^{-1})}{A(q^{-1})} q\ u(k) + \qquad (4.5\text{-}60)$$
$$+ \frac{L(z^{-1})}{R_d(z^{-1})\ D(z^{-1})} \left[y(k) - \frac{B(q^{-1})}{A(q^{-1})} q^{-d}\ u(k) \right]$$

which can be transformed using identity (1.1-58) into

$$y_a(k+d+1/k) = \frac{L(q^{-1})}{R_d(q^{-1})D(q^{-1})} y(k) + \frac{C(q^{-1})F(q^{-1})qB(q^{-1})}{A(q^{-1})\ D(q^{-1})} u(k) \qquad (1.1\text{-}61)$$

The unpredictable part of the auxiliary output corresponds to the unpredictable part of the noise component, so

$$y_a(k+d+1) = y_a(k+d+1/k) + F(q^{-1})v(k+d+1) \qquad (1.1\text{-}62)$$

Using this equation the cost function I_3 can be expressed as

$$I_3 = E\left\{[y_a(k+d+1/k) - w_a(k)]^2 + u_a^2(k)\right\} + E\left\{F(q^{-1})\ v(k+d+1)\right\} \qquad (1.1\text{-}63)$$

Minimization of the I_3 with respect to $u(k)$ yields

$$\frac{\partial I_3}{\partial u(k)} = 2[y_a(k+d+1/k) - w_a(k)] \frac{\partial y_a(k+d+1)}{\partial u(k)} + 2u_a(k) \frac{\partial u_a(k)}{\partial u(k)} = 0 . \qquad (1.1\text{-}64)$$

Since the polynomials $C(z^{-1})$, $F(z^{-1})$, $A(z^{-1})$ and $D(z^{-1})$ are monic, it follows from Eqns.(1.1-61), (1.1-55)

$$\frac{\partial y_a(k+d+1)}{\partial u(k)} = b_1 , \qquad (1.1\text{-}65)$$

$$\frac{\partial u_a(k)}{\partial u(k)} = r_o \qquad (1.1\text{-}66)$$

respectively and Eq.(1.1-64) can be transformed using Eqns. (1.1-55), (1.1-56) and (1.1-61) into

$$\left[\frac{L(q^{-1})}{D(q^{-1})R_d(q^{-1})}\,y(k) + \frac{C(q^{-1})F(q^{-1})qB(q^{-1})}{A(q^{-1})D(q^{-1})}\,u(k) - \right.$$

$$\left. - R_w(q^{-1})w(k)\right]b_1 + r_o R_u(q^{-1})u(k) = 0\ , \tag{1.1-67}$$

yielding finally the control law

$$u(k) = \frac{R_w(q^{-1})A(q^{-1})D(q^{-1})R_d(q^{-1})w(k) - L(q^{-1})A(q^{-1})y(k)}{C(q^{-1})F(q^{-1})R_d(q^{-1})qB(q^{-1}) + \frac{r_o}{b_1}\,D(q^{-1})R_d(q^{-1})A(q^{-1})R_u(q^{-1})} \tag{1.1-68}$$

This is the generalized minimum variance controller, which minimizes the cost criterion (1.1-53) and all other minimum variance controllers, given earlier in this chapter are its special cases. Some interpretations of this control law are given in Gawthrop (1977) and in the work of Clarke in Harris, Billings (1981).

g) Minimum variance controllers for disturbances with nonzero mean

A nonzero mean disturbance $E(\ n(k)\ ) \neq 0$ is produced if the noise filter has a pole at $z = 1$, which causes a drifting disturbance. If such disturbances are acting on a proportional process, an integral type controller is preferable in order to avoid steady-state control errors. Since a integral type noise filter and a proportional type process are of interest in this section, only MV1-d and MV2-d controllers will be discussed.

Due to the integral character of the noise filter ($C(1) = 0$) the MV2-d controller in this case is of the integral type also, comp.Eq. (1.1-24). In the case of the MV1-d controller an integral type controller can be obtained by adding an integral acting term, see Isermann (1981, 1985). This practical but rather heuristic approach has also a theoretical background, which will be discussed next.

The optimization criterion Eq (1.1-3) is not suitable in the case of nonzero mean disturbances, since u(k) tends to a nonzero value as t tends to the infinity. So the optimization criterion is modified to the following form

$$E \{ y^2(k+d+1) + r\, \Delta u^2(k) \} \tag{1.1-69}$$

where $\Delta u(k)$ is the difference of the manipulated variable

$$\Delta u(k) = u(k) - u(k-1) \; . \tag{1.1-70}$$

Now the MV1-d controller is designed for the modified process

$$G_P'(z) = \frac{y(z)}{\Delta u(z)} = \frac{1}{1 - z^{-1}} G_P(z) = \frac{B(z^{-1})}{(1 - z^{-1})\, A(z^{-1})} z^{-d} \tag{1.1-71}$$

and the desired controller is obtained using Eq. (1.1-70) and (1.1-25)

$$G_R(z) = \frac{u(z)}{e(z)} = \frac{\Delta u(z)}{(1 - z^{-1})\, e(z)} = \tag{1.1-72}$$

$$= \frac{1}{(1 - z^{-1})} \; \frac{(1 - z^{-1})\, A(z^{-1})\, L(z^{-1})}{zB(z^{-1})C(z^{-1})\, F(z^{-1}) + \frac{r}{b_1} (1 - z^{-1})A(z^{-1})D(z^{-1})} =$$

$$= \frac{A(z^{-1})\, L(z^{-1})}{zB(z^{-1})C(z^{-1})F(z^{-1}) + \frac{r}{b_1} (1 - z^{-1})A(z^{-1})D(z^{-1})}$$

This controller is of integral type since C(1) = 0 and neither $A(z^{-1})$ nor $L(z^{-1})$ have a zero at z = 1. For $A(z^{-1})$ this is obvious since a proportional process was supposed, for $L(z^{-1})$ this can be shown by the following contradiction:

If $L(z^{-1})$ would have a zero at $z = 1$, then the right hand side of the identity (1.1-18) would have a zero at $z = 1$. Consequently also the left hand side of this identity, i. e. $D(z^{-1})$ would have a zero at $z = 1$, which would cancel the zero of $C(z^{-1})$ and the noise filter would not be of the integral type.

Other possibility to eliminate steady state control error is to estimate the DC value and to compensate it by a separate feedback.

1.2 Generalized predictive controller

The generalized predictive controller, Clarke et all, (1987a,b), is the successor of generalized minimum variance controller, described in section 1.1. The optimal process output prediction interpretation is enriched by the ideas of predictive control, Richalet et all, (1978) and dynamic matrix control, Cutler, Ramaker, (1980). Though various extensions of the generalized predictive controller are possible, it will be represented here in its original form, which uses the process model represented in Fig. 1.1 with the noise model polynomials

$$D(z^{-1}) = 1 \tag{1.2-1}$$

and

$$C(z^{-1}) = (1-z^{-1})\, A(z^{-1}) = \Delta\, A(z^{-1}) \tag{1.2-2}$$

where

$$\Delta = 1-z^{-1} \tag{1.2-3}$$

With these two assumptions the process is described by

$$y(z) = \frac{B(z^{-1})}{A(z^{-1})}\, z^{-d}\, u(z) + \frac{1}{\Delta\, A(z^{-1})}\, v(z) \tag{1.2-4}$$

and as drifting disturbances are modeled, the resulting controller will be of the integral type (see discussion in section 1.1g), what is a very practical solution.

The idea of the generalized predictive controller is to predict the process output not only for d steps as minimum variance controller does, but for a set of steps, and to minimize the cost function

$$I = E\left\{ \sum_{i=n_1}^{n_2} [y(k+i) - w(k+i)]^2 + \sum_{i=1}^{n_u} r(i)\, [\Delta u(k+i-1)]^2 \right\} \tag{1.2-5}$$

where w is the reference signal.

The first term of this cost function penalizes the system error $e(k)=w(k)-y(k)$ on a prediction horizon stretching from n_1 to n_2, where n_1 and n_2 are so called minimum and maximum costing horizons respectively. The second term of Eq. (1.2-5) takes into account the manipulated variable increments $\Delta u(k)$ rather then the manipulated variables themselves. The reason for this is the offset in the manipulated variable signal which is necessary for the systems with no integrating properties to have zero steady state offset. The upper limit n_u represents the so called control horizon, i. e. the maximum number of manipulated variable increments; while the increments beyond n_u are supposed to be zero

$$\Delta u(k+i) = 0 \qquad \text{for } i \geq nu \tag{1.2-6}$$

The weighting factor r(i) can be variable to penalize the manipulated variable increments differently, but for simplicity a constant r(i) = r will be used.

According to the cost function (1.2-5) a set of process output predictions should be calculated. A prediction for i steps is computed analogously as in chapter 1.1, i. e.: the prediction of the noise component is

$$n(k+i) = \frac{L(q^{-1})}{\Delta\ A(q^{-1})}\ v(k) + L(q^{-1})\ v(k+i) \tag{1.2-7}$$

where the polynomial $L(q^{-1})$ of order m and the monic polynomial $F(q^{-1})$ of order i-1 are determined from the Diophantine identity

$$1 = \Delta\ A(q^{-1})\ F(q^{-1}) + L(q^{-1})q^{-i} \tag{1.2-8}$$

The optimal noise prediction corresponds again to the first term in Eq. (1.2-7) and since v(k) can be calculated from the past values of the controlled and manipulated variables, it is given by

$$n(k+i/k) = L(q^{-1}) \left[y(k) - \frac{B(q^{-1})}{A(q^{-1})} q^{-d} u(k) \right] . \qquad (1.2\text{-}9)$$

The optimal process output prediction is given by the following expression

$$y(k+i/k) = \frac{B(q^{-1})}{A(q^{-1})} q^{-(d-i)} u(k) + n(k+i/k) , \qquad (1.2\text{-}10)$$

which can be written using Eqns. (1.2-8) and (1.2-9) as

$$y(k+i/k) = q^{-d}B(q^{-1})F(q^{-1})\Delta u(k+i) + L(q^{-1})y(k) . \qquad (1.2\text{-}11)$$

Predictions according to this equation should be calculated for i going from n_1 to n_2 and in each step Eq. (1.2-8) should be solved for $L(q^{-1})$ and $F(q^{-1})$. The corresponding successive solutions can be obtained recursively, Clarke et all, (1987a) as follows: Denoting the solution of the Diophantine equation (1.2-8) for the i-th step as $L^i(q^{-1})$ and $F^i(q^{-1})$, then the solution for the i+1st step is obtained according to

$$F^{i+1}(q^{-1}) = F^i(q^{-1}) + q^{-i} l_o^i \qquad (1.2\text{-}12)$$

$$L^{i+1}(q^{-1}) = q\left[L^i(q^{-1}) - \Delta A(q^{-1})\right] l_o^i \qquad (1.2\text{-}13)$$

where l_o^i is the first coefficient of the polynomial $L^i(q^{-1})$. The initial conditions for the recursions are obtained for i = 1 according to

$$F^1(q^{-1}) = 1 \qquad (1.2\text{-}14)$$

$$L^1(q^{-1}) = q\left[1 - \Delta A(q^{-1})\right] \qquad (1.2\text{-}15)$$

Denoting

$$H^i(q^{-1}) = q^{-d}B(q^{-1})F^i(q^{-1}) = \qquad (1.2\text{-}16)$$

$$= h^i_{d+1}q^{-(d+1)} + h^i_{d+2}q^{-(d+2)} + \ldots + h^i_{d+m+i-1}q^{-(d+m+i-1)}$$

the set of optimal process output predictions can be written as follows

$$y(k+1/k) = h^1_{d+1}\,\Delta u(k-d) + \ldots + h^1_{d+m}\,\Delta u(k-d-m+1) + \\ + l^1_o\,y(k) + \ldots + l^1_m\,y(k-m)$$

$$\vdots$$

$$y(k+d/k) = h^d_{d+1}\,\Delta u(k-1) + \ldots + h^d_{2d+m-1}\,\Delta u(k-d-m+1) \\ + l^d_o\,y(k) + \ldots + l^d_m\,y(k-m)$$

$$y(k+d+1/k) = h^{d+1}_{d+1}\,\Delta u(k) + \ldots + h^{d+1}_{2d+m}\,\Delta u(k-d-m+1) \\ + l^{d+1}_o\,y(k) + \ldots + l^{d+1}_m\,y(k-m)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$y(k+d+n/k) = h^{d+n}_{d+1}\,\Delta u(k+n-1) + \ldots + h^{d+n}_{d+n}\,\Delta u(k) + h^{d+n}_{d+n+1}\,\Delta u(k-1) + \\ + \ldots + h^{d+n}_{2d+n+m-1}\Delta u(k-d-m+1) + l^{d+n}_o\,y(k) + \ldots + l^{d+n}_m\,y(k-m)$$

$$(1.2\text{-}17)$$

The cost function Eq. (1.2-5) involves only predictions between the minimum and maximum costing horizons n_1 and n_2 respectively. As $y(k+d+1/k)$ is the first prediction being influenced by the current manipulated variable increment, a reasonable choice for the minimum costing horizon is $n_1=d+1$, however an underestimation of the process dead time and the corresponding minimum costing horizon is not critical. The maximum costing horizon should be set approximately to the rise time of the process. Large values of the control horizon n_u allow the control signal to be active for a long time and consequently to behave like deadbeat control, while small values provide smooth control. Further suggestions about the choice of costing and control horizons the reader may find in, Clarke et all, (1987a), where 1, 10 and 1 for n_1, n_2 and n_u respectively are proposed for typical industrial processes.

Due to the Eq. (1.2-12) the first i-1 terms of the polynomial $F^{i+1}(q^{-1})$ are equal to the polynomial $F^i(q^{-1})$ and the same is valid for polynomials $H^{i+1}(q^{-1})$ and $H^i(q^{-1})$ respectively, so

$$
\begin{aligned}
h^1_{d+1} &= h^2_{d+1} = \dots = h^{i-1}_{d+1} = h_{d+1} \\
h^1_{d+2} &= h^2_{d+2} = \dots = h^{i-2}_{d+2} = h_{d+2} \\
&\vdots \\
h^i_{d+i-1} &= h_{d+i-1}
\end{aligned}
\tag{1.2-18}
$$

and as considering Eqns. (1.2-16) and (1.2-8) the polynomial $H(q^{-1})$ can be expressed in the following form

$$
H^i(q^{-1}) = \frac{B(q^{-1})}{\Delta\, A(q^{-1})}\, q^{-d}\left[1 - L(q^{-1})q^{-i}\right] , \tag{1.2-19}
$$

the first i-1 terms h^i represent the first i-1 nonzero step responses of the process transfer function.

The optimal process output predictions (1.2-17) are based on the current and past values of the process output and on the past, current and future values of the manipulated variable increments. The current value of the process output and the past values of the process output and the manipulated variable increments respectively are known at the time k, the current and the future values of the manipulated variable increments are to be determined in order to minimize the cost function (1.2-5).

Considering Eqns. (1.2-6) and (1.2-18), the optimal process output predictions (1.2-17) can be written for $n_1=d+1$ in the vector form, Schumann (1989)

$$\underline{y}^+ = \underline{H}\,\underline{u}^+ + \underline{P}\,\underline{u}^- + \underline{L}\,\underline{y}^- \qquad (1.2\text{-}20)$$

where $\underline{y}^+$ is the vector of the predicted process outputs

$$\underline{y}^+ = [y(k+n_1/k) \quad y(k+n_1+1/k) \quad \ldots \quad y(k+n_2/k)]^T , \qquad (1.2\text{-}21)$$

$\underline{y}^-$ the vector of the present and the past process outputs

$$\underline{y}^- = [y(k) \quad y(k-1) \quad \ldots \quad y(k-m)]^T , \qquad (1.2\text{-}22)$$

$\underline{u}^+$ the vector of the current and the future manipulated variable increments

$$\underline{u}^+ = [\Delta u(k) \quad \Delta u(k+1) \quad \ldots \quad \Delta u(k+n_u-1)]^T , \qquad (1.2\text{-}23)$$

$\underline{u}^-$ the vector of the past manipulated variable increments

$$\underline{u}^- = [\Delta u(k-1) \quad \Delta u(k-2) \quad \ldots \quad \Delta u(k-d-m+1)]^T , \qquad (1.2\text{-}24)$$

$\underline{H}$ the $(n_2-n_1) \times n_u$ matrix of those coefficients h_j^i, which represent the process step response and are denoted according to (1.2-18) by h_j

$$\underline{H} = \begin{bmatrix} h_{d+1} & 0 & \cdots & 0 \\ h_{d+2} & h_{d+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ h_{d+nu} & h_{d+nu-1} & \cdots & h_{d+1} \\ \vdots & \vdots & & \vdots \\ h_{d+n_2-n_1} & h_{d+n_2-n_1-1} & \cdots & h_{d+n_2-n_1-n_u+1} \end{bmatrix} \tag{4.6-25}$$

$\underline{P}$ the $(n_2-n_1) \times (m+d-1)$ matrix consisting of the remaining coefficients h_j^i

$$\underline{P} = \begin{bmatrix} h^{n_1}_{n_1+d+1} & h^{n_1}_{n_1+d+2} & \cdots & h^{n_1}_{n_1+m+2d-1} \\ h^{n_1+1}_{n_1+d+2} & h^{n_1+1}_{n_1+d+3} & \cdots & h^{n_1+1}_{n_1+m+2d} \\ \vdots & \vdots & & \vdots \\ h^{n_2}_{n_2+d+1} & h^{n_2}_{n_2+d+2} & \cdots & h^{n_2}_{n_2+m+2d-1} \end{bmatrix} \tag{1.2-26}$$

and $\underline{L}$ the $(n_2-n_1) \times m$ matrix consisting of the polynomial $L^i(q^{-1})$ elements

$$\underline{L} = \begin{bmatrix} l^{n_1}_o & l^{n_1}_1 & \cdots & l^{n_1}_m \\ l^{n_1+1}_o & l^{n_1+1}_1 & \cdots & l^{n_1+1}_m \\ \vdots & \vdots & & \vdots \\ l^{n_2}_o & l^{n_2}_1 & \cdots & l^{n_2}_m \end{bmatrix} \tag{4.7-27}$$

Introducing the vector of the future reference signals

$$\underline{w} = [w(k+n_1) \quad w(k+n_1+1) \quad \ldots \quad w(k+n_2)]^T \tag{1.2-28}$$

and noting that $e(y(k+i)) = y(k+i/k)$, the cost function (1.2-5) can be now written in the vector form

$$I = [\underline{H}\ \underline{u}^{+} + \underline{P}\ \underline{u}^{-} + \underline{L}\ \underline{y}^{-} - \underline{w}]^{T}\ [\underline{H}\ \underline{u}^{+} + \underline{P}\ \underline{u}^{-} + \underline{L}\ \underline{y}^{-} - \underline{w}] + r\ \underline{u}^{+T}\underline{u}^{+}\ . \tag{1.2-29}$$

The minimization of I results in the control algorithm

$$\underline{u}^{+} = [\underline{H}^{T}\underline{H} + r\ \underline{I}]^{-1}\ \underline{H}^{T}\ [\underline{w} - \underline{P}\ \underline{u}^{-} - \underline{L}\ \underline{y}^{-}] \tag{1.2-30}$$

and as for the control in the moment k only the first element of the vector $\underline{u}^{+}$, i.e. $\Delta u(k)$ is needed, the control law (1.2-30) may be written in standard form

$$(1-z^{-1})\ P(z^{-1})\ u(z) = F(z)\ w(z) - Q(z^{-1})\ y(z) \tag{1.2-31}$$

where the coefficients of the polynomial $P(z^{-1})$ are determined by the first row of the matrix $[\underline{H}^{T}\underline{H} + r\ \underline{I}]^{-1}\underline{H}^{T}\ \underline{P}$

$$[p_1\quad p_2\quad \ldots\quad p_{m+d-1}] = [1\quad 0\quad \ldots\quad 0]\ [\underline{H}^{T}\underline{H} + r\ \underline{I}]^{-1}\underline{H}^{T}\ \underline{P} \tag{1.2-32}$$

and the coefficients of the polynomial $Q(z^{-1})$ by the first row of the matrix $[\underline{H}^{T}\underline{H} + r\ \underline{I}]^{-1}\underline{H}^{T}\ \underline{L}$

$$[q_0\quad q_1\quad \ldots\quad q_m] = [1\quad 0\quad \ldots\quad 0]\ [\underline{H}^{T}\underline{H} + r\ \underline{I}]^{-1}\underline{H}^{T}\ \underline{L} \tag{1.2-33}$$

The prefilter polynomial F(z) is given by

$$F(z) = f_{n_1} z^{n_1} + f_{n_1+1} z^{n_1+1} + \ldots + f_{n_2} z^{n_2} \tag{1.2-34}$$

where the coefficients f_i are determined by

$$[f_{n_1} \quad f_{n_1+1} \quad \cdots \quad f_{n_2}] = [1 \quad 0 \quad \ldots \quad 0] \, [\underline{H}^T\underline{H} + r\,\underline{I}]^{-1}\underline{H}^T \, . \tag{1.2-35}$$

The realization of the term F(z)w(z) requires the knowledge of the future reference signals.

Generalized predictive controller design results in a standard linear controller, but with a noncausal prefilter, which takes into account the future reference signals. The orders of the controller polynomials $P(z^{-1})$ and $Q(z^{-1})$ are $(m+d)$ and m respectively and are equal to the orders obtained by the pole placement design. Some other properties of generalized predictive controller and its relations with other controllers the reader may find in, Clarke, Mohtadi, (1987).

2 SUPERVISION AND COORDINATION

The application of parameter-adaptive controllers requires all assumptions used for the derivation of the parameter-adaptive control principle, the criteria for parameter estimation and controller design to be met and some design parameters to be chosen properly.

At the real processes control these a priori assumptions are violated due to expected or unexpected changes in the operating conditions of the controlled process or adaptive control loop. This may result in unacceptable or unstable control behaviour of the parameter-adaptive controller. A continuous monitoring of several functions of the parameter-adaptive control loop is therefore required.

The first work in this direction was performed by Clarke and Gawthrop(1981), Schumann et.al.(1981), Fortescue et. al. (1981), Bergmann (1983), Isermann and Lachmann (1985). The supervision and necessary coordination tasks entailed in this monitoring can be realized as a third level feedback.
The supervision and coordination level involves tasks for recognition of faulty functions, diagnosis, of reasons and initiation of remedial measures and incorporates functions for coordination of the start-up procedure, decision making as to when a new set of controller parameters will be used for control, decision making as to which set of process model parameters will be taken for controller parameter calculation, filtering of estimated process parameters, monitoring of the parameter estimation and controller design procedure to recognize faulty functions of the algorithms and to take appropriate actions to avoid influence on the control behaviour, supervision of the closed loop stability and stabilization of the control loop in the case of unstable parameter-adaptive controller, general improvement of the control behaviour.

For the design of the supervision and coordination level it is necessary to eliminate, identify or reduce the causes of faulty functioning of the parameter estimation procedure,controller design procedure or close loop behaviour.

In most of the applications the faulty function of the parameter-adaptive controller can be traced back to violations of assumed pre-conditions. These are divided into "violations to be overcome" and "violations not to be overcome".

"Violations not to be overcome" are those that cannot be removed by the supervision and coordination level. These types of violations are similar to the problems encountered by fixed controllers in the case of very fast changes of process dynamics, unstable structure of controller/process combinations or, in the case of adaptive controllers lack of persistent excitation of the process input signal all the time.

The unfavorable effects on the performance of the parameter-adaptive control loop can be reduced or eliminated by the supervision and coordination level. This third feedback level can include additional functions for improvement of the control behaviour and for operational ease.

It should be mentioned that a general supervision and coordination levels do not exist. For each application the realization of the on-line supervision and coordination (S&C)-level is unique and depend on the main goal, effort, available computation time,intentions and overall supervisory philosophy.

The supervisory philosophy can consist of an active influence in the control loop, as in consideration of estimation uncertainties and generation of an additional test signal in the case of suboptimal dual controllers or a passive influence, for example to use the controller parameter set and to replace the actual controller parameter set only when the estimated process parameters describe the process with sufficient accuracy. Therefore, a decision must be made to adopt either active or passive influence when considering the design of the S&C-level.

2.1 Start-up procedure - pre identification and model verification

The parameter-adaptive control loop can be started in closed loop after specifying the free parameters and setting the initial conditions for parameter estimation and controller design. In this case the initial process model estimates available for the controller design procedure are either non existent or have poor confidence, which may result in undefined control action.

This control action mainly depends on the type of controller, the initial state of the process and the external disturbances acting on the process output signal. To avoid large variations of the process input signal, this signal can be limited by software-boundaries. However it cannot be guaranteed that the resulting process input signal is persistently exciting for the process model parameter estimation to match the process behaviour accurately enough.

In order to avoid large or possibly unacceptable process input and output signals within the S & C - level, a pre-identification phase is employed. This is done open loop for stable process dynamics; and closed loop with a fixed controller for unstable process dynamic.

During this pre-identification phase the process is perturbed with a sufficient exciting input signal, to satisfy the identifiability conditions in order to estimate the parameters of the process model and obtain a reasonable starting model in open loop. After a sufficient identification time the estimated process model is verified, by checking the identified model behaviour against the real process input/output behaviour.

This comparison of process output and process model output signal can be achieved automatically or by an operator in a visual manner on a terminal screen. As identification is generally an iterative procedure, this method is also applicable to the start-up procedure, where different design parameters are used for parameter estimation.

This pre-identification can also include an on-line search method for the structure of the process model, such as is described in Schumann et. al.(1981).

Before closing the parameter-adaptive control loop a robust - back-up controller (P, PI, PID-controller depending on estimated process model) is calculated for the identified process model and the operating point used for pre-identification.

2.2 Supervision of parameter estimation

The main aim of supervising the parameter estimation is to ensure that the identified process model adequately matchable dynamic input/output behaviour of the real process. For model verification, the same methods as would be used for open loop identification can be applied in principle for the identification procedure of the S & C - level and should also operate on-line and in real time. In addition to model verification carried out in the supervision and coordination of the parameter estimation, other actions will be necessary such as parameter estimation switch-off, filtering of estimates, restart of parameter estimation and change of free specific estimation parameters.

Monitoring of estimation signals:

According to the parameter estimation method used, i.e. recursive least squares method(RLS), extended least-squares method(RELS) or the square root version of the least squares method(SRF) the following quantities are available for rating the present state of the estimation algorithm:

a) signal values:

The behaviour of these quantities have been investigated in Bergmann (1983), Lachmann(1983), Radke(1984), Isermann and Lachmann(1986) for different cases of process and control loop conditions such as change of process dynamics, varying noise signals, non-persistent excitation, etc. All these simulations and experiments have shown that there is no unique quantity that could be used to identify an erroneous parameter estimation procedure.

Only the combined performance of several quantities allows the possibility of monitoring a faulty function of the estimator, classifying the reason for errors and taking appropriate actions to improve the parameter estimation.

No persistent excitation:

In the closed parameter-adaptive control loop the variations of the process input and output signal can become rather small in the case of a well tuned controller, thus reducing changes of the reference signal in the case where no external disturbances are exciting the control loop.

Therefore, for the parameter estimation algorithm, no useful information about the process dynamics can be gained from the measured process input and output signal values. The signals for identification are not persistently exciting signals, and problems may arise for continued parameter estimation particularly if a fading memory is chosen. In this case linear dependent rows will appear in the information matrix, such that the identification problem is unsolvable, as in the case of a variable forgetting factor as proposed by Fortesque et al. (1981), Anderson (1985) or Kosut et. al. (1987).

Using the recursive version of least squares or extended least squares this circumstance is indicated by an increasing variance of all parameter estimates, which drift to wrong values known as bursting of parameter estimates and a divergence of the elements or trace of the inverse of the information matrix. These erroneous parameter estimates may generate unacceptable or unstable control behaviour. For the square root estimation method, the information matrix, which is normally given in a triangular form, shows zero rows and the algorithms is not numerically solvable.

A simple action to avoid influence on the parameter estimates is an automatic switch-off of the identification procedure. For the square root estimation method it is very simple to prove the positive definiteness of the information matrix according to the triangular form. The equations of the unique parameter estimation algorithms must also be solved to causality of the identification procedure.

Fast changes of process properties:

The parameter estimation may have difficulties to identify the correct values of a linear process model if there are fast changes of the real process properties (gain, time constant) or large and fast changes of the operating point in a highly nonlinear process. This problem will not arise if the forgetting factor ischosen very small. But in the case of external disturbances, mostly valid for real processes, small forgetting factors produce an estimated process model for the disturbance behaviour and adapts the parameter-adaptive controller for this disturbance model. A forgetting factor between 0.7 and 1.0 results in a process model,which doesn't match the present operation conditions of the control loop. The estimated model describes a not valid dynamic behaviour and is insufficient for controller design. The unacceptable process model parameters are indicated by the a priori error, particularly by its variance.

Within the S & C - level it is possible to monitor the error value and its variance. Additionally for the square root estimation algorithm the value of the loss function V(k), which unavoidably is a result of the estimation procedure.

For very fast or large changes in the process properties it is even possible that a reduced forgetting factor (fading, but infinite memory) and large values of the a priori error e(k) will adopt the parameter estimates too slowly.
As mentioned above, for a forgetting factor $\lambda(k) < 1.0$ and no changes of the reference signal for a large time interval and an external noise signal acting on the process output the estimation procedure adapts to the process noise behaviour. To avoid this change of the estimated parameters caused by strong external disturbances, the forgetting factor. $\lambda(k)$ has to be increased if there is no change of the reference signal during a sequence of sampling intervals.

Unstationary and periodic disturbances:

External disturbances such as step changes, drift signals or impulses acting on the process output signal used for parameter estimation are violations of the pre-conditions for the parameter estimation procedure. These disturbances lead to a drastic change of the estimated process parameter which do not result from changes of the process dynamics and cause a deterioration of the control behaviour. This is also valid for a periodic disturbance signal, with an eigenfrequency within the bandwidth of the process.

The estimated values react to the given disturbances with a step change and return to their original values after several sampling intervals depending on value of the forgetting factor. Even using the explicit dc-value-estimation, the influence on the process parameter estimates can be reduced but not eliminated.

Differentiation of the estimation equation results in a delta-impulse in the derived estimation equation for a step change in the process output signal. This explicit dc-value estimation only can guarantee, that the estimated parameters return to these original values. When unstationary and periodic disturbances occur, this is indicated in the estimated parameters variance. If this fault is recognized the parameter estimates are already changed and the only thing to be done is to wait till the estimated parameters are returned back to their original values. During that time no controller calculation should be performed. A far better solution is to interfere in the parameter estimation procedure before the parameter estimates are be influenced.

Therefore it is proposed that the measured values of the process output signal be evaluated by a signal regression analysis method, before they are used for parameter estimation. By such a procedure it impossible to recognize with high probability step changes, drift signals, impulses signals and periodic disturbances in the process output signal within a few sampling intervals.

In the case of output signal values which do not conform with the past values, the parameter estimation and dc-value estimation is switched off.

In spite of all different supervisory functions for parameter estimation, in the case of undefined influences on the parameter-adaptive control loop an unstable divergent parameter estimating is possible.

This arises mainly for real, complex processes, which cannot be approximated well enough by a process model.

For a divergent parameter estimation which can be observed in a monotonically increasing value of the error mean and its variance the error is no longer a white noise signal.

A convergent estimation is also not given if the trace of the variance matrix P is not monotonically decreasing. In such a situation a "restart" of the parameter estimation algorithm is necessary.

If a temporary worsening of the control performance is acceptable, and a restart of the parameter estimation is considered as too strong also a sufficiently exciting test signal can be added to the process input signal (cautious adaptive controllers) combined with a reduction of the forgetting factor λ.

For the parameter-adaptive control of a process where a change of the process model structure (order m or dead time d) is expected, an additional on-line search for the structure of the process model should be activated. This on-line search for the structure parameters can be performed in parallel for a given time interval without indicating a variation of the supervision quantities.

Filtering of process signals and parameter estimates:

Experiments with parameter-adaptive controllers relating to the behaviour of parameter estimation in the presence of a stochastic noise signal have shown that, especially for a forgetting factor $\lambda(k)<1$, the parameter estimates vary considerably. These persistent variations of the estimates which do not result from changes in the process dynamics give an unsettled behaviour of the manipulated signal and complicate the analysis of the supervisory quantities. A considerable reduction in the parameter variations can be achieved by filtering the process input and output signal switch the same discrete low pass filter, or with an optimal Bessel- or Butterworth filter.

Additional filtering of the parameter estimates used for the process model, supervisory quantities and controller design, results in a substantially smoother behaviour of the parameter estimates and thus in less variations of the controller parameters and manipulated variable.

The same filters used for the input/output signals can be used for filtering of the parameter estimates. However, recursive filter algorithms with finite memory are to be preferred, because of changing of the parameter steady-states and very low frequencies.

2.3 Supervision of controller design

In addition to the supervision of the parameter estimation procedure it is important to supervise and coordinate the controller design. The procedure of controller design in the basic parameter-adaptive control loop, where a new controller parameter calculation is performed in each sampling interval based on the present estimated process model, seems to be unsuitable.

Controller calculation at time instants defined by the S & C - level improves the security of the parameter-adaptive control loop, decreases the computation effort and reduces the influence of fast and frequent controller parameter changes on the control loop. In order to avoid the application of controller types where the pre-conditions for the estimated process model are violated, these pre-conditions can be checked. Further,more a different sample time for control and parameter estimation is possible within the S & C -level.

Violations of pre-conditions:

Typical violations of pre-conditions for controller design are incompatible combinations of process structure and controller structure or cancellations of process poles or process zeros near to or outside of the unit-circle of the Z-plane in the discrete case. For example; cancellations of process poles by the deadbeat controller or cancellations of process zeros for the minimum variance controller. Therefore due is a danger of instability of the parameter-adaptive control loop, because for a real process a precise pole/zero cancellation is not possible.

The process model poles and zeros are calculated for each controller design valid process model, and only if all pre-conditions for the controller design procedure used, are fulfilled, a new set of controller parameters is determined.

Frequent and fast parameter calculation/changes:

Changes in the dynamic of the real process result in a variation of the estimated process model parameters and yield a transient phase from one steady-state parameter set to another.During this phase the estimated process parameters should not be used for controller design, because it cannot be guaranteed that the process model matches the real process dynamics and that the resulting control loop.

The duration of the transient phase, which depends on the value of the forgetting factor is detectable by observing the variance of signals of the parameter estimates. If the parameter estimates have reached their new stationary values the variance values are again within an ϵ-band. Variations within this ϵ-band are the effects of "normal" variations of the parameter estimates due to stochastic disturbances and numerical effects of the process computer as discussed in the previous section. This ϵ-band can be small, if filtering of the process signals and the parameter estimates is realized.

When the variance values of parameter estimates are placed within the ϵ-band a calculation of the controller parameters based on the present process model can be performed. By filtering of the process input and output signals and the estimated process model parameter the variation of the process model parameters can be so small, that only a single controller design is needed. The next new controller design is only required after a change. of the process model parameters, when the variance values of the process model parameters leave the ϵ -band for the stationary mean value value. This is an indication that the process model has changed and a new controller design is necessary.

To monitor very slow drifting parameter changes, which cannot be realized by the calculation of the mean values and variance values, the trend of the parameter estimates and the mean values of the parameter estimates has to be considered, and if they are monotonously increasing or decreasing then the controller parameters have to be calculated continuously.

For the supervision and coordination of the previous discussed methods it is not necessary to consider all parameter estimates. Experiments have shown that 2 to 4 parameters of the process model are sufficient.

Different sampling time for controller design:

For many of the control algorithms which are well suited to parameter-adaptive control the choice of an appropriate sampling time is critical with respect to the control action due to set point changes or external disturbances. A characteristic feature of the most used control algorithms is, that the smaller the sample time the stronger the control action. This control action must however be acceptable with respect to actuator constraints. On the other hand, the choice of a sampling time suitable for parameter estimations possible within a relatively wide range. The choice of the sample time for parameter-adaptive control is mainly complicated by the applied control algorithm. One possible solution to this problem is the use of different sampling times for parameter estimation and control. A priori only the sampling time for parameter estimation is chosen relatively small (within the range appropriate for parameter estimation) and on-line the sampling time for control is adapted in order to obtain acceptable control.

The on-line adaptation of the control sampling time is performed recursively by transformation of the estimated discrete time process model to a model at a multiple sample time. This may be achieved by simulation of the closed loop control behaviour for a given change in the reference value and the evaluation of a defined loss function or performance index, resulting in a sampling time factor i that can vary between 1 and a given maximum value. A very simple index for the control action is the controller transfer function parameter q_0 which is equal to the first value of the control signal after a unit step disturbance to the controller input. This can be assumed to take smaller values as the sample time is increased.

Unacceptable controller action:

In the case when the controller action of the implemented controller is to weak or to strong the weighting factor for the manipulated variable should be changed off-line by the operator, because the choice of the weighting factor and the rate of the controller action is based on engineering experience and difficult to computerize in a performance index.

When it is necessary to change the controller type, this should also be initiated off-line by the operator, the S & C -level can provide a complete simulation of the control loop behaviour with the designed new controller type and the present estimated process model. The actual process loop conditions, or generated changes of the reference signal of the simulated control loop can also be provided in the case of an "off-line" change of the controller weighting factor r.

For the supervision of the closed loop, especially the stability of the closed parameter-adaptive control loop, the behaviour of the manipulated variable and the controlled signal is analyzed.If the manipulated variable is at the maximum or minimum boundary for a given number of control steps and the control deviation $e(k) = w(k)-y(k)$ is monotonically increasing or oscillatory for a constant reference signal, the parameter - adaptive controller is switched back to the "back-up" controller designed during the pre-identification phase, and the reference signal is changed to the values given by the operation point for pre-identification phase.

3 COMPARISON OF PARAMETER-ADAPTIVE CONTROLLERS BY SIMULATIONS

The parameter-adaptive control systems were tested by simulation of the test process IX, Isermann (1987), i.e. by the second order process

$$G_p(z) = \frac{0.01866928\ z^{-1} + 0.01746399\ z^{-2}}{1 - 1.782598\ z^{-1} + 0.8187308\ z^{-2}} \tag{3-1}$$

which is the discrete time equivalent of the continuous time process

$$G_p(s) = \frac{y_p(s)}{u(s)} = \frac{K}{1 + 2TDs + T^2s^2} \tag{3-2}$$

for K = 1, D = 0.5, T = 5s and a sampling time T_o = 1s. Note that this is the same process as used in section 4.4 of 1987 Report, Matko, Schwamberger 1987 for testing model reference adaptive control systems.

The parameter-adaptive control systems were tested in deterministic and stochastic environment. Two noise filters were used in the stochastic case

$$G_{n1}(z) = \frac{1}{A(z^{-1})} = \frac{1}{1 - 1.782598\ z^{-1} + 0.8187308\ z^{-2}} \tag{3-3}$$

and

$$G_{n2}(z) = \frac{D(z^{-1})}{A(z^{-1})} = \frac{1 - 1.559573\ z^{-1} + 0.03700681\ z^{-2}}{1 - 1.782598\ z^{-1} + 0.8187308\ z^{-2}} \tag{3-4}$$

and the standard deviation of the noise n(k) was chosen to be 0.04. The process and noise filter parameters were supposed to be unknown and they were identified by U-D factorisation modification of the recursive least squares (RLS) and recursive extended least squares (RELS) methods.

The used control algorithms were minimum variance controllers MV4 and MV3 with $r = 0.001$ and the generalized predictive controller (GPC) with $n_1 = 1$, $n_2 = 10$, $n_u = 1$, $r = 0$ and $r = 1$ respectively for the stochastic case and pole placement with integral characteristic (LC PA), perfect linear model following (PLMFC), extended order deadbeat (DB $\nu+1$) and the generalized predictive controller (GPC) with $n_1 = 1$, $n_2 = 10$, $n_u = 1$ and $r = 0$ for the deterministic case respectively. With pole placement design the four poles were placed at $0.779786 \pm i\ 0.177418$ and with the perfect linear model following controller the desired transfer function was chosen to

$$G_w(z) = \frac{0.04296456\ z^{-1} + 0.03700681\ z^{-2}}{1 - 1.559573\ z^{-1} + 0.6395440\ z^{-2}}\ . \qquad (3\text{-}5)$$

Note, that the chosen poles of the pole placement design correspond to the desired transfer function poles and the reference model with reference model adaptive control systems, Part C.

The purpose of the simulation was to illustrate and evaluate different control schemes, so "pure" adaptive control without initial identification, supervision and coordination was applied and the controller parameters were changed in each step according to the identified process parameters. The methods were compared without and with the reference signal excitation. As excitation an asymmetric pulse signal with the period of 100 s

$$\begin{aligned} w(t) &= 1 \qquad (n * 100 + 70)\ s > t > n * 100\ s \\ w(t) &= 0 \qquad (n+1)\ 100\ s > t > (n * 100 + 70)s \end{aligned} \qquad (3\text{-}6)$$

$$n = 0, 1, 2, \ldots$$

was applied. In Fig. 3.1 typical time responses of the normalized parameter errors are shown for identification in closed loop with a GPC controller and with reference signal excitation. The normalized parameter error of the process transfer function denominator

$$\tilde{a}_1 = \frac{a_1 - \hat{a}_1}{a_1} \tag{3-7}$$

is represented for identification schemes without noise and with the noise filters 1/A and D/A respectively. With noise filter D/A also the normalized parameter error of the noise filter numerator

$$\tilde{d}_1 = \frac{d_1 - \hat{d}_1}{d_1} \tag{3-8}$$

is shown.

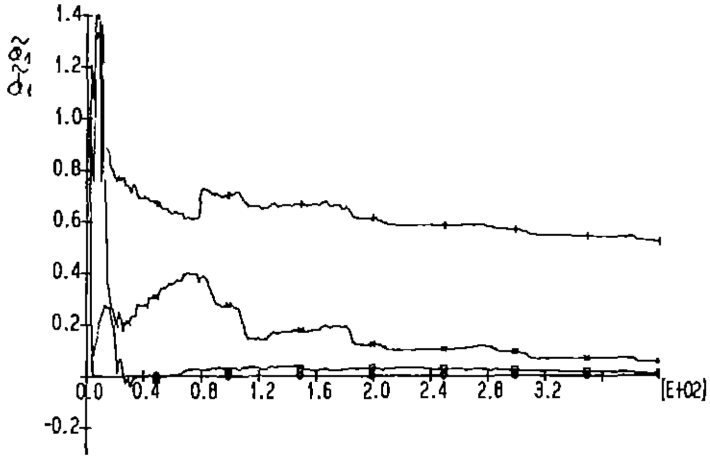

Fig. 3.1: The normalized parameter errors

deterministic case: o - $\tilde{a}_1$

stochastic case - noise filter 1/A: # - $\tilde{a}_1$

noise filter D/A: x - $\tilde{a}_1$

\+ - $\tilde{d}_1$

Tab. 3-1 evaluates the identification performance criterions for different controllers. The following criterions were used for the comparison:

- The sum of normalized parameter errors:

$$I_1 = \frac{1}{n} \sum_{i=1}^{n} \sum_{k=0}^{20000} \frac{\theta_i - \hat{\theta}_i(k)}{\theta_i} \tag{3-9}$$

where θ_i denotes process parameters a_i and b_i and n is the number of identified parameters corresponding to 4 for noise filter 1/A and 6 for noise filter D/A respectively.

- The sum of identification equation errors

$$I_2 = \sum_{k=0}^{20000} e(k)^2$$

Tab. 3-1: The identification performance criterions

Controller	Noise filt.	Numb. param.	External excitation	I_1	I_2
GPC	no noise	4	yes	3.9	0.0097
MV4	1/A	4	no	14.8	0.435
GPC	1/A	4	no	54.5	0.435
MV3	D/A	6	no	53471.	39.7
GPC	D/A	6	no	11397.	29.6
MV4	1/A	4	yes	5.4	0.498
GPC	1/A	4	yes	8.3	0.448
MV3	D/A	6	yes	4949.	56.5
GPC	D/A	6	yes	2808.	31.1

The identification of the parameters with noise filter 1/A is much faster than with the noise filter D/A and the denominator polynomial parameters a_i are better identified than the noise filter numerator polynomial parameters d_i. The left side of Fig. 3.2 represents the process output without control (i.e. the applied noise) and with four controllers for the stochastic case. The corresponding manipulated variable signals are shown in the right side of Fig. 3.2. The process output and the manipulated variable variances are illustrated in Tab. 3-2 where also the variances with the exact controllers are shown.

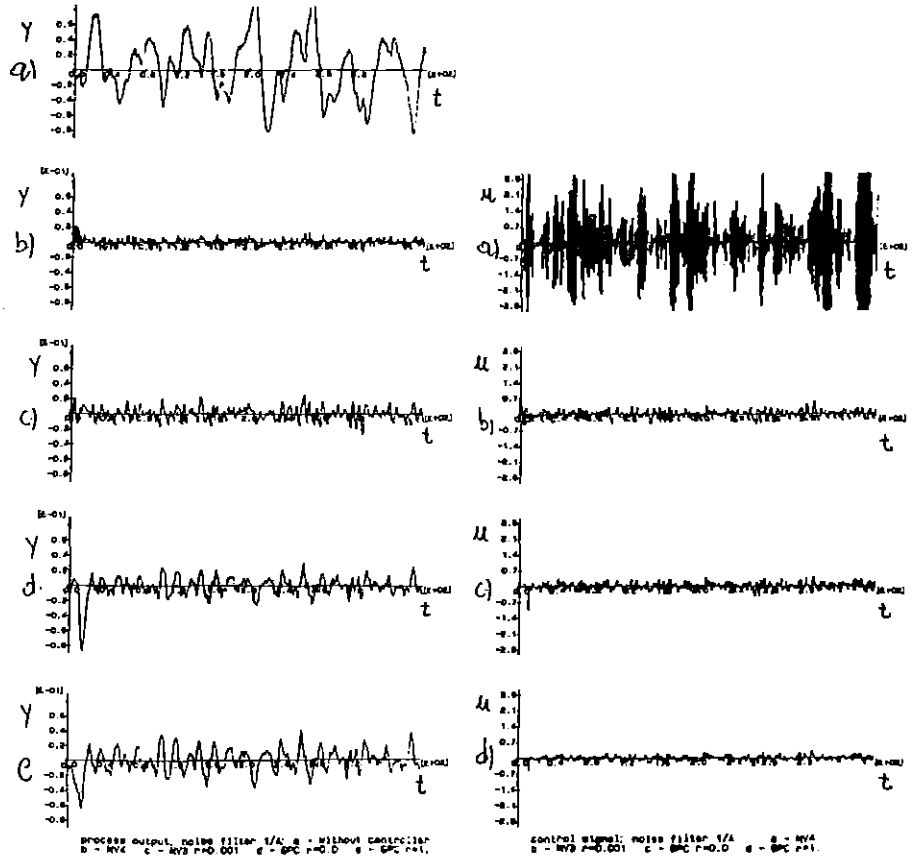

Fig. 3.2: The process output and the corresponding manipulated variable signal for the stochastic case:
- a - no control
- b - MV4
- c - MV3 (r=0.001)
- d - GPC (r=0)
- e - GPC (r=1)

Tab.3-2 The process output and the manipulated variable variances

Controller	Noise filter	r	Parameters	$\sigma_y^2 * 10^{-5}$	$\sigma_u^2 * 10^{-3}$
MV4	1/A	0.0	exact	2.03	1770.
GPC	1/A	0.0	exact	7.60	23.8
MV4	1/A	0.0	identified	2.42	2440.
MV4	1/A	0.001	identified	6.93	23
GPC	1/A	0.0	identified	18.5	21.5
GPC	1/A	1.0	identified	24.7	9.77
MV3	D/A	0.0	exact	122.2	2527.
GPC	D/A	0.0	exact	142.2	18.2
MV3	D/A	0.0	identified	937148.	3274272.
MV3	D/A	0.001	identified	145.3	42.3
GPC	D/A	0.0	identified	391.8	449.2
GPC	D/A	1.0	identified	237.2	180.5

Minimum variance controllers, especially MV4, minimize the process output variance very good, but the price for that is a very high control effort, which is very hard to realize in practice. Generalized predictive controllers, with a little bit greater process output variance seemed to be better applicable

Fig. 3.3 represents the process output and manipulated variable signals with three controllers for the deterministic case. The applied reference signal was the asymmetric pulse signal given by Eq. (3-6) and the adaptation was started at t=100s. The corresponding control performance criterions

$$I_3 = \sum_{k=300}^{399} [y_p(k) - w(k)]^2 \qquad (3\text{-}10)$$

$$I_4 = \sum_{k=300}^{399} [u(k) - U_{oo}(k)]^2 \qquad (3\text{-}11)$$

where U_{oo} is the steady-state value of the manipulated variable, are illustrated in Table 3-3 where also criterions for the deadbeat controller with original and increased orders are given.

Tab. 3-3:The control performance criterions

Controller	I_3	I_4
GPC	8.91	1.74
PLMFC	7.71	5.35
LC PA	5.40	39.5
DB(ν+1)	3.52	741.5
DB(ν)	2.47	2450.0

The deadbeat controllers, which are actually designed to reach steady state in 2 and 3 steps (seconds) respectively, what is a very short time, have the control effort, which can not be realized in the practice. The deadbeat controllers are given in the Tab. 3 for illustration purposes only; in the practice such controllers can be applied on low pass processes with large sample times. The results of other controllers are very similar, but with PLMFC a suitable prescribed transfer function and with LC PA a careful selection of prescribed closed loop poles is necessary.

The simulation has shown, that the GPC may be applied with an appropriate choice of design parameters n_1, n_2 and n_u in the deterministic and stochastic case. Minimum variance controllers in the stochastic case are applicable only with manipulated variable weighting (r>0). In the deterministic case linear controllers with pole placement and perfect linear model following controllers can be implemented with an appropriate choice of the desired poles, while deadbeat controllers are applicable only with increased order and large sampling times on low pass processes.

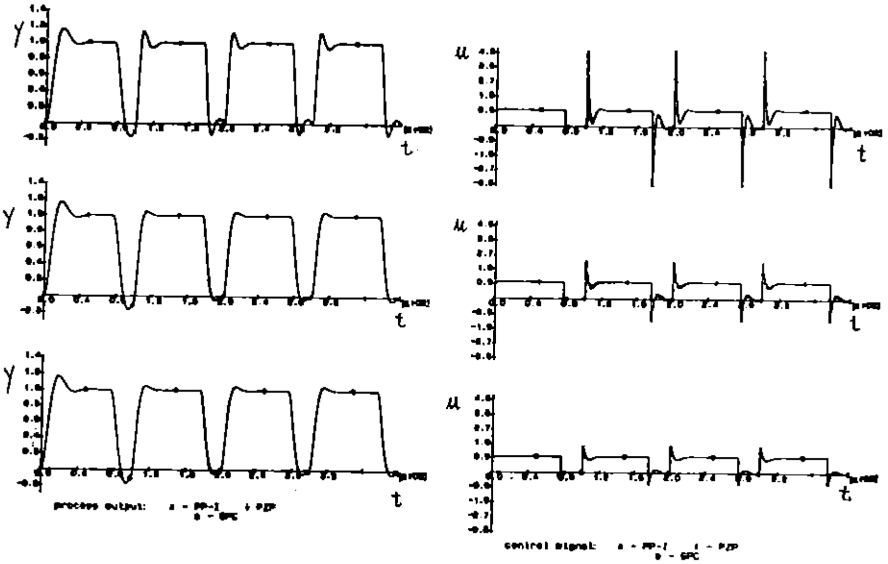

Fig.3.3: The process output and the corresponding manipulated variable signals for the deterministic case: a - LC PA

b - PLMFC

c - GPC

4 ADAPTIVE CONTROL OF THE PILOT PLANT

The recursive least squares algorithm and generalized predictive controller were tested in the parameter - adaptive control loop on a laboratory pilot plant. In designing the pilot plant the following guidelines were considered:

- it should represent a simple laboratory model of a hypothetical process plant, often used in industry,
- from the system theoretical viewpoint it should be a multivariable system consisting of a few subsystems with a moderate number of interrelated systems variables,
- as much as possible available standard process control components should be included to allow similarity with real process plants and on the other side to keep investment costs within reasonable limits,
- possibilities for either local or remote control should be provided,
- a convenient degree of flexibility of various configurations was sought with provisions for easy modifications or extensions,
- from the education point of view it should represent a plant providing basic exercises on the courses of modeling and control.

Keeping these guidelines in mind, the plant was designed in the form of a laboratory process system conceptually consisting of a hydraulic and a thermal part. Fig. 4.1 represents the scheme of the pilot plant.

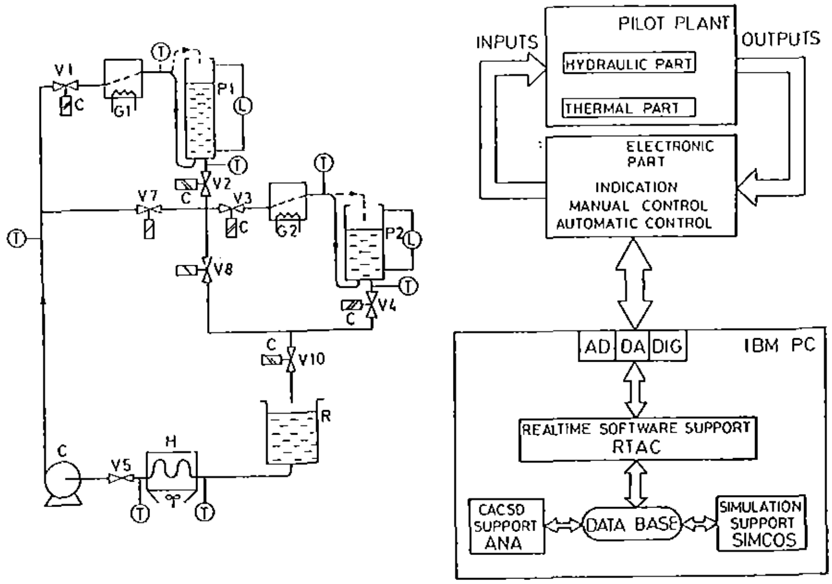

Fig.4.1. The scheme of the laboratory pilot plant

The symbols in the scheme have the following meaning:

C	circulation pump
G1,G2	electric heaters
P1,P2	level tanks
R	reservoir for liquid storage
H	cooler
V1,V2,V3,V4,V10	electromagnetic control valves
V5	manually operated valve
V7,V8	electromagnetic on/off valves

The hydraulic part of the plant consists of two vertical cylindrical glass tanks in which the levels of the liquid can be measured by level transducers. The level tanks are connected with each other and with an auxiliary liquid-storage reservoir in the closed circulation circuit system which also contains a circulation pump, a number of piping cross connections and remotely controllable electromagnetic valves. The thermal part of the plant is superposed onto the hydraulic part simply by inclusion of two electrical heaters and cooler into the piping system. In this way the working liquid can be heated and also cooled to near normal room temperature. The sensors and actuators mounted in the plant were selected from cheaper standard production programs for process instrumentation. They include:

- two capacity-type liquid level sensors,
- five resistance-type (Pt-100) temperature probes,
- two diffused-silicon differential pressure transducers,
- eight electromagnetic control valves,
- two standard 2 kW electrical heaters.

The pilot plant is complemented by a separate power and control cabinet containing:

- indication with displays of all measuring variables,
- potentiometers for manual control of valves and heaters,
- electronics for transducers and transmitters,
- electronics for controlling the electromagnetic valves, ventilator, pump and heaters (thyristor regulators),
- galvanic isolation of all measuring signals,
- connectors with analog and digital signals for process computer connection,
- power supply, power switches, fuses, protection relays.

The hydraulic-thermal plant was connected to process IBM PC computer workstation through the control cabinet. The process hardware enables sampling of all measured signals using A/D converters and controlling of valves, heaters, pump and cooler using D/A converters and digital outputs (DIG).

Only hydraulic part (two tanks) was used in our experiments and only a few experiments were performed. The purpose of the experiments was to test the hardware and software for pilot plant control and to implement a simple adaptive control loop. Because of its good performance in the simulation tests the generalized predictive controller and recursive least squares method were chosen as the controller and identification procedure respectively. The aim of the experiments was to test the robustness of the overparametrization of the plant model and the sensitivity of the adaptive control loop to the sampling time. The results are shown in Figs 4.2 to 4.13. In Fig. 4.2 to 4.4 the adaptive control with sampling time 10 sec. and two parameter estimation is shown. The parameters of the controller design were $r = 0.5$, $n_u = 1$, $n_1 = 1$, $n_2 = 10$ and the supposed delay was one sample. Figs. 4.2, 4.3 and 4.4 show the controlled variable, the control variable and the estimated parameters respectively.

In Fig. 4.5 to 4.7 the adaptive control with sampling time 10 sec. and four parameter estimation is shown. The parameters of the controller design were $r = 0.5$, $n_u = 1$, $n_1 = 1$, $n_2 = 10$ and the supposed delay was one sample. Figs. 4.5, 4.6, 4,7 and 4.8 show the controlled variable, the control variable and the estimated a and b parameters respectively.

In Fig. 4.9 to 4.11 the adaptive control with sampling time 4 sec. and two parameter estimation is shown. The parameters of the controller design were $r = 0.5$, $n_u = 1$, $n_1 = 1$, $n_2 = 10$ and the supposed delay was one sample. Figs. 4.8, 4.9 and 4.10 show the controlled variable, the control variable and the estimated parameters respectively.

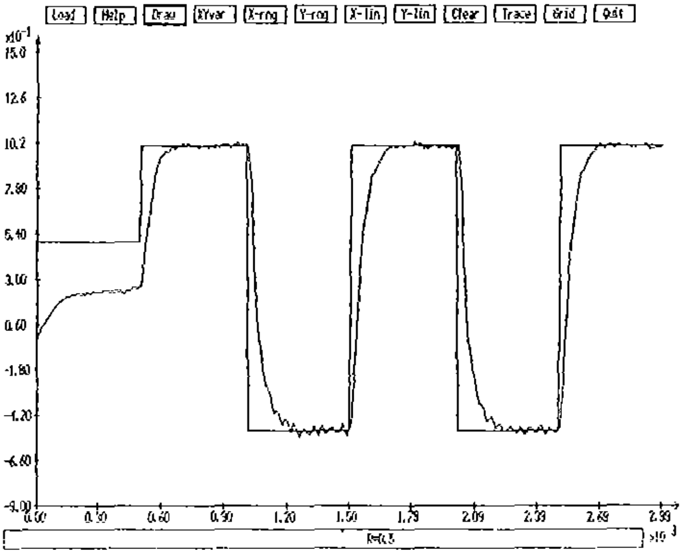

Fig. 4.2 Controlled variable with T_s=10 sec and 2 parameters

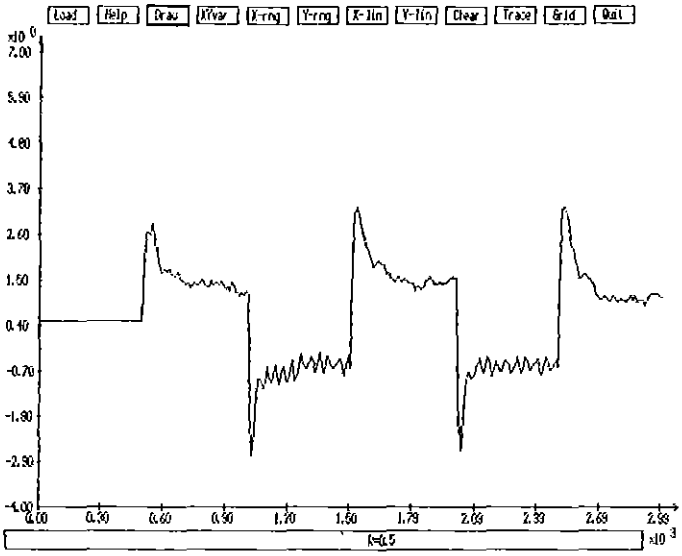

Fig. 4.3 Control variable with T_s=10 sec and 2 parameters

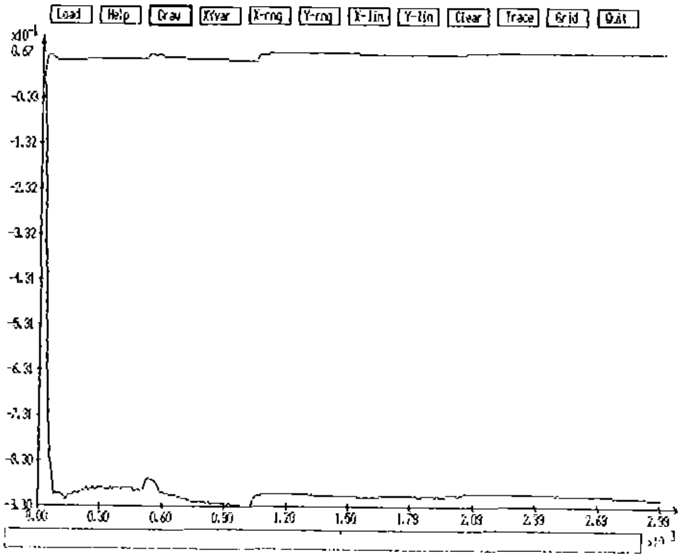

Fig. 4.4 Estimated parameters with T_s=10 sec and 2 parameters

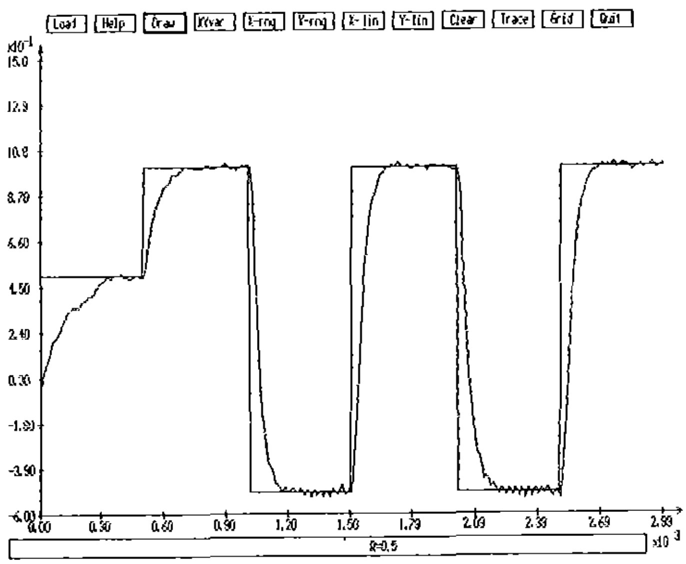

Fig. 4.5 Controlled variable with T_s=10 sec and 4 parameters

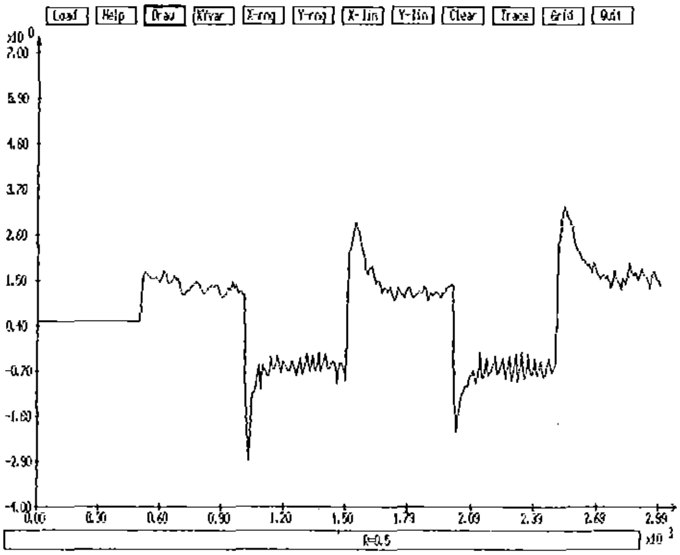

Fig. 4.6 Control variable with T_s=10 sec and 4 parameters

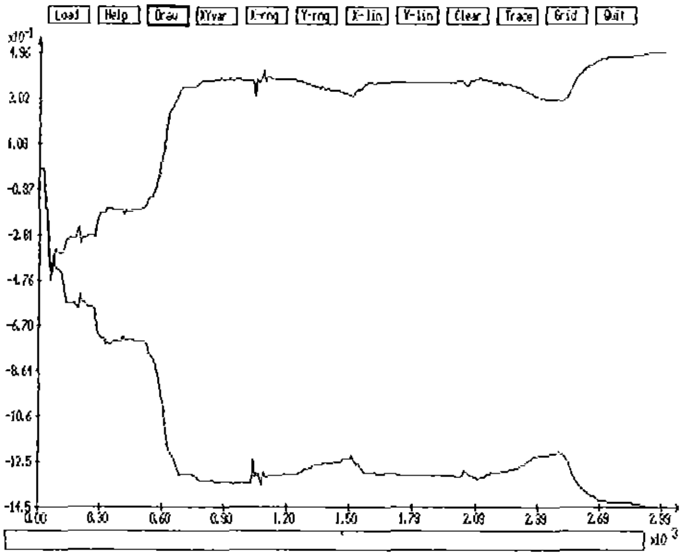

Fig. 4.7 Estimated a parameters with T_s=10 sec and 4 parameters

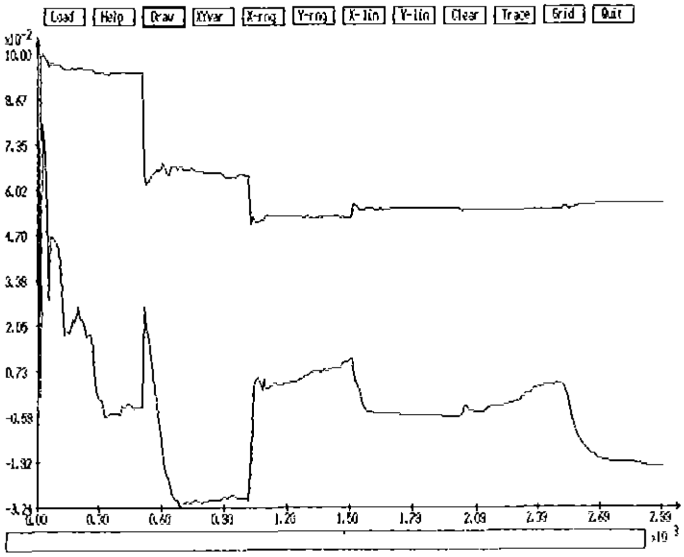

Fig. 4.8 Estimated b parameters with T_s=10 sec and 4 parameters

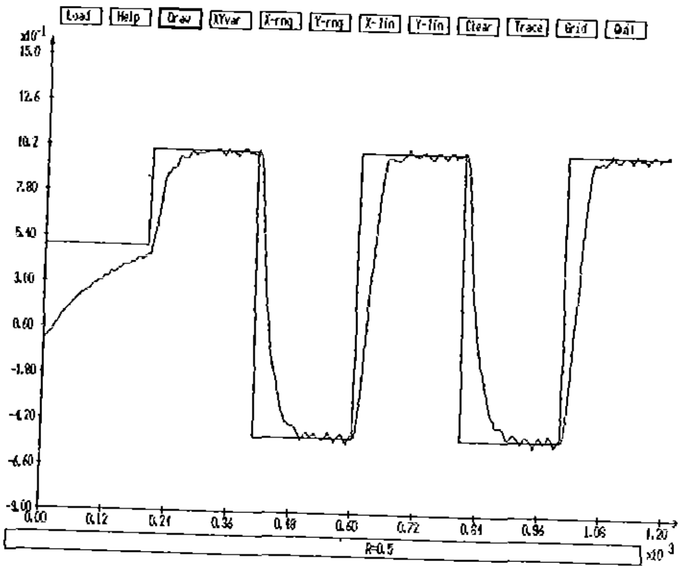

Fig. 4.9 Controlled variable with T_s=4 sec and 2 parameters

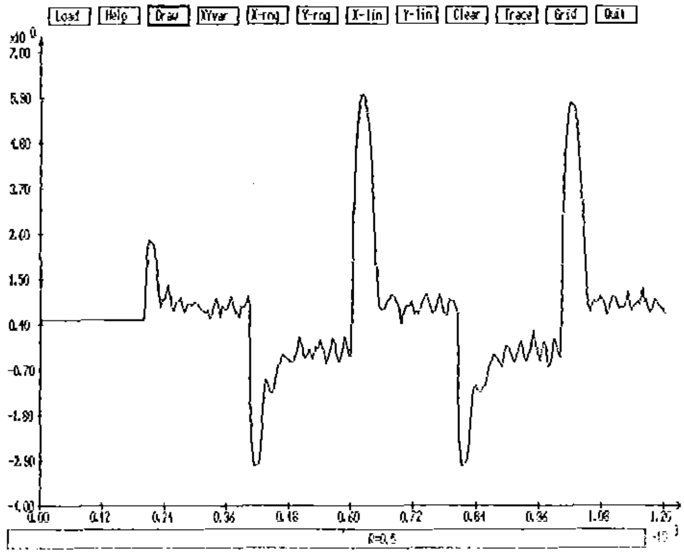

Fig. 4.10 Control variable with T_s=4 sec and 2 parameters

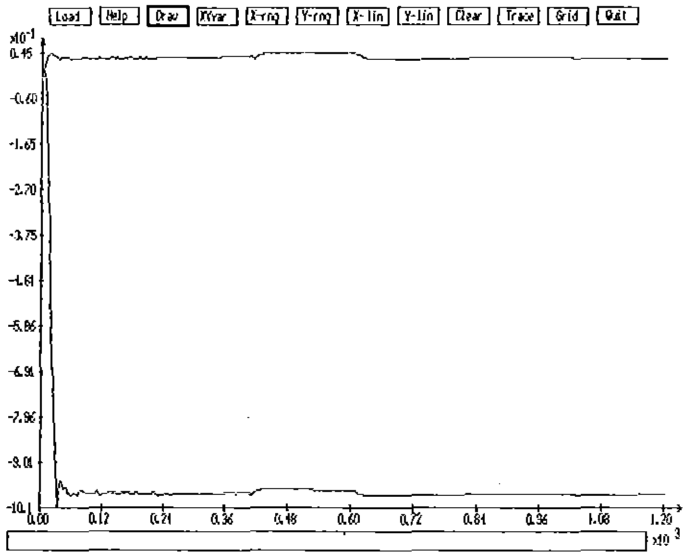

Fig.4. 11 Estimated parameters with T_s=4 sec and 2 parameters

In Fig. 4.12 to 4.14 the adaptive control with sampling time 20 sec. and two parameter estimation is shown. The parameters of the controller design were r = 0.5, $n_u = 1$, $n_1 = 1$, $n_2 = 10$ and the supposed delay was one sample. Figs. 4.12 4.13 and 4.14 show the controlled variable, the control variable and the estimated parameters respectively.

The experiments have shown that the adaptive control algorithm is in the practice not sensitive to parameter overparametrization and to the sampling time, however that large control variables occur with small sampling times. This means that only the sampling time has to be chosen carefully in the practice. further experiments are in the progress.

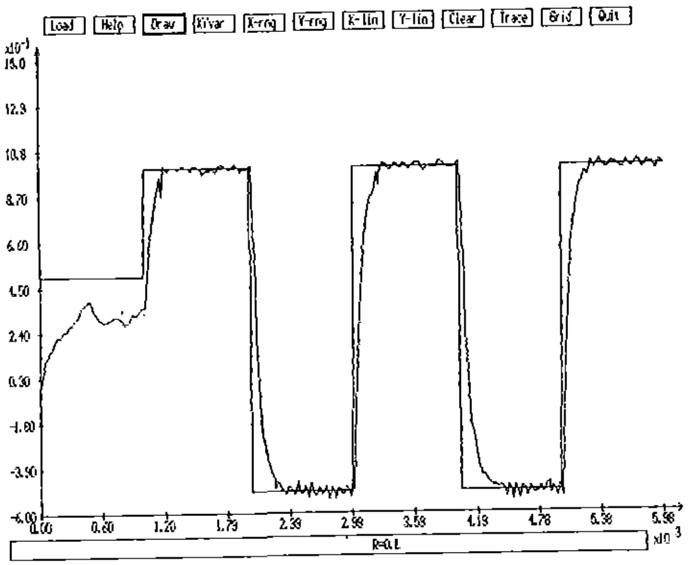

Fig. 4.12 Controlled variable with T_s=20 sec and 2 parameters

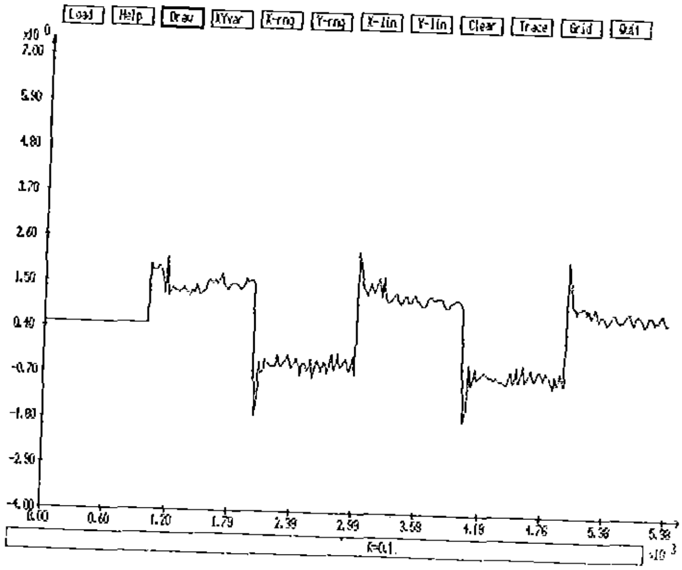

Fig. 4.13 Control variable with T_s=20 sec and 2 parameters

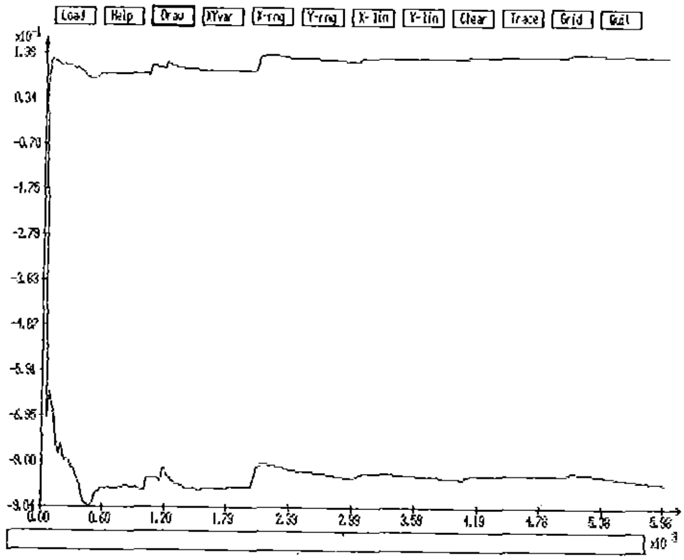

Fig. 4.14 Estimated parameters with T_s=4 sec and 2 parameters

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